

**MATH 2001 DISCRETE MATHEMATICS  
WEEK 9 QUIZ  
2026 MARCH 13**

PROBLEM S1-1

Let  $A = \{1, 2, 6, 7, 10\}$  and let  $B = \{x \in \mathbb{R} \mid 4 \leq x \leq 8\}$ . Find the least element of  $A \cap B$ . (Show some work to explain your answer.)

PROBLEM S1-2

Which set has a larger cardinality:  $\{n \in \mathbb{N} \mid 50 \leq n \leq 75\}$  or  $\mathcal{P}(\{1, 2, 3, 4, 5\})$ ? (Show some work to explain your answer.)

PROBLEM S2-1

Define  $f: \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = \frac{1}{x^2+1}$ . Is  $f$  injective, surjective, both, or neither? Explain your reasoning.

PROBLEM S2-2

Define  $g: \mathbb{N} \rightarrow \mathbb{N}$  by the rule  $g(n) = n + 5$ .

- a. Find the image of  $g$ . (You do not need to list all of its elements. You can just use set-builder notation or otherwise precisely specify the form of its elements.)
- b. Let  $Y = \{n \in \mathbb{N} \mid n \leq 10\}$ . List the elements of  $g^{-1}(Y)$ .

PROBLEM S3-1

Use a truth table to demonstrate that

$$(\neg P \wedge Q) \vee (P \wedge \neg Q) \equiv \neg(P \wedge Q) \wedge (Q \vee P).$$

Show intermediate steps, not just the final result.

PROBLEM S3-2

If we know that  $P \rightarrow Q$  is true and  $Q \rightarrow R$  is false, can we conclude that  $P \rightarrow R$  is false? Explain your reasoning.

PROBLEM S4-1

Let  $n \in \mathbb{N}$ . Prove that if  $3^n$  is a perfect square then  $n$  is even.

PROBLEM S4-2

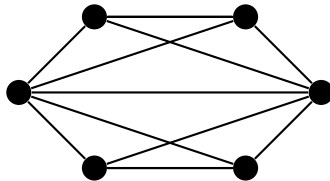
Suppose that  $f: \mathbb{N} \rightarrow \mathbb{N}$  has the property that  $f(n+1) \leq f(n)$  for each  $n \in \mathbb{N}$ . Prove that the image of  $f$  is a finite set.

## PROBLEM S5-1

Suppose that a graph has 7 vertices, which have degrees 5, 4, 3, 3, 3, 2, and 2. How many edges must the graph have?

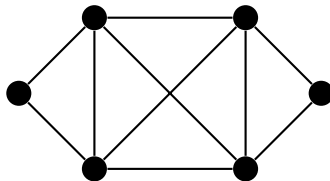
## PROBLEM S5-2

Determine whether the following graph is planar. If it is planar, give a planar drawing of it. If it is non-planar, explain how you know that.



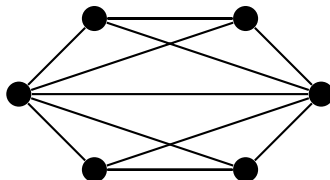
## PROBLEM S6-1

Determine the chromatic number of the following graph. Show your reasoning, don't just give a coloring with no explanation.



## PROBLEM S6-2

Determine the chromatic number of the following graph. Show your reasoning, don't just give a coloring with no explanation.



## PROBLEM S7-1

Let  $R \subset \mathbb{N}^2$  be given by

$$R = \{ (x, y) \in \mathbb{N}^2 \mid x > 0 \text{ and } y > 0 \}.$$

If  $R$  is an equivalence relation on  $\mathbb{N}$ , prove that. Otherwise, explain why it is not.

## PROBLEM S7-2

Suppose that  $\sim$  is an equivalence relation on a set  $A$  and that  $f: A \rightarrow B$  is a function. Must it be the case that

$$\{ (f(x), f(y)) \in B^2 \mid x \sim y \}$$

is an equivalence relation on  $B$ ? Explain your reasoning.