MATH 2001 DISCRETE MATHEMATICS
WEEK 6 QUIZ
2026 FEBRUARY 20

Problem S1-1

Find the greatest element of

$$\left\{ x \in \mathbb{N} \;\middle|\; -x^2 + 7x - 10 > 0 \right\}.$$

(Show some work to explain your answer.)

Problem S1-2

Let $S = \{2, 4, 6\}$. Compute the cardinality of

$$\{ A \in \mathcal{P}(S) \mid |A| \text{ is even} \}$$

(Show some work to explain your answer.)

Problem S2-1

Let $\mathbb{R}_{\geq 0} = \{ x \in \mathbb{R} \mid x \geq 0 \}$ and define $f\colon \mathbb{R}_{\geq 0} \to \mathbb{R}$ by $f(x) = x^4 + 1$. Is f injective, surjective, both, or neither? Explain your reasoning.

Problem S2-2

Let $A = \{0, 1, 2, 3\}$ and let $B = \{1, 3, 4, 9, 16, 25, 27, 81\}$. Define $g\colon A \to B$ by the rule $g(x) = 3^x$.

a. Find the image of g.

b. Let $Y = \left\{ b \in B \;\middle|\; b = n^2 \text{ for some } n \in \mathbb{N} \right\}$. List the elements of $g^{-1}(Y)$.

Problem S3-1

Use a truth table to demonstrate that

$$P \wedge (Q \to \neg P) \equiv \neg(P \to Q).$$

Show intermediate steps, not just the final result.

Problem S3-2

Suppose that $P \to Q$ is false. Can we conclude that $\neg P \to \neg Q$ in this situation? Explain why or why not.

Problem S4-1

Suppose that $n = a^3 + 1$ for some $a \in \mathbb{N}$. Prove that if n is odd then a is even.

Problem S4-2

Suppose that $f\colon \mathbb{N} \to \mathbb{N}$ is injective and that $g\colon \mathbb{N} \to \mathbb{N}$ is also injective. Prove that $g \circ f\colon \mathbb{N} \to \mathbb{N}$ is injective.