MATH 2001 DISCRETE MATHEMATICS
WEEK 12 QUIZ
2026 APRIL 10

Problem S1-1

Let $A = \{\, n \in \mathbb{N} \mid 5 \le 2^n \le 50 \,\}$ and let $B = \{1, 3, 8, 25\}$. Find the greatest element of $A \cup B$. (Show some work to explain your answer.)

Problem S1-2

Compute the cardinality of $\mathcal{P}(\mathcal{P}(\mathcal{P}(\varnothing)))$. (Show some work to explain your answer.)

Problem S2-1

Define $f\colon \mathbb{N} \to \mathbb{N}$ by $f(n) = n^n - 1$. Is f injective, surjective, both, or neither? Explain your reasoning.

Problem S2-2

Define $g\colon \{3, 5, 7, 11, 13\} \to \mathbb{N}$ by the rule $g(n) = n^2$.

a. Find the image of g.

b. Let $Y = \{\, n \in \mathbb{N} \mid n + 1 \text{ is a multiple of } 5 \,\}$. List the elements of $g^{-1}(Y)$.

Problem S3-1

Use a truth table to demonstrate that

$$P \to (P \to Q) \equiv \neg(P \land \neg Q).$$

Show intermediate steps, not just the final result.

Problem S3-2

If we know that $P \to Q$ is true and $Q \to P$ is true, can we conclude that P is true? Explain your reasoning.

Problem S4-1

Let $m, n \in \mathbb{N}$. Show that if $m^2 + n^2$ is odd then either m is odd or n is odd.

Problem S4-2

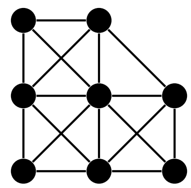
Prove that if $f\colon \mathbb{R} \to \mathbb{R}$ is a surjective function then the function $g\colon \mathbb{R} \to \mathbb{R}$ given by $g(x) = (f(x))^3$ is also surjective.

Problem S5-1

Suppose that a graph has 7 vertices, which have degrees 4, 4, 4, 3, 3, 2 and 2. How many edges must the graph have?

Problem S5-2

Determine whether the following graph is planar. If it is planar, give a planar drawing of it. If it is non-planar, explain how you know that.

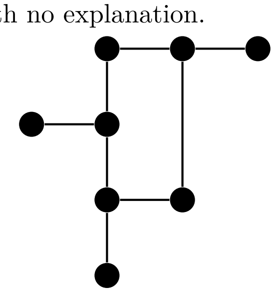


Problem S6-1

Determine the chromatic number of the graph from Problem S5-2 above. Show your reasoning, don't just give a coloring with no explanation.

Problem S6-2

Determine the chromatic number of the following graph. Show your reasoning, don't just give a coloring with no explanation.



Problem S7-1

Let $R \subset \mathbb{N}^2$ be given by

$$R = \left\{ (x, y) \in \mathbb{N}^2 \;\middle|\; x = y \text{ or } x + y \text{ is odd} \right\}.$$

If R is an equivalence relation on $\mathbb{N}$, prove that. Otherwise, explain why it is not.

Problem S7-2

Suppose that R and S are both equivalence relations on a set A. Prove that $R \cap S$ is also an equivalence relation on A.

Problem S8-1

What is the coefficient of x^5 in $(x+2)^8 + (x+3)^9$?

Problem S8-2

How many elements of the set $\{ n \in \mathbb{N} \mid 1 \le n \le 600 \}$ are multiples of 6, 10, or 15?