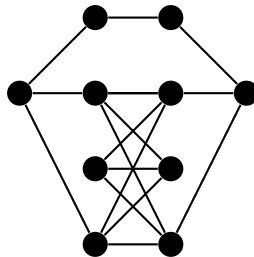


**2001 DISCRETE MATHEMATICS
HOMEWORK 5
DUE 2025 MARCH 29**

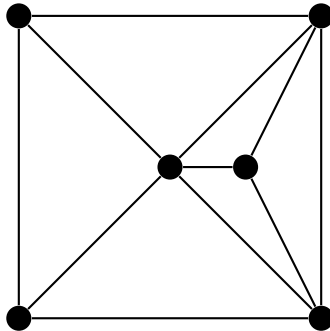
PROBLEM 1 (S6, 2 POINTS)

Determine the chromatic number of the following graph. Show your reasoning, don't just give a coloring with no explanation.



PROBLEM 2 (S7, 2 POINTS)

Determine the chromatic **index** of the following graph. (Remember, the chromatic index is like the chromatic number for edge coloring.) Show your reasoning, don't just give a coloring with no explanation.



PROBLEM 3 (S7, 3 POINTS)

Let $R \subset A^2$ be a relation on a set A . We say that R is *cyclic* when $x R y$ and $y R z$ imply that $z R x$. Prove that if R is both reflexive and cyclic then R is an equivalence relation. Is every equivalence relation cyclic?

PROBLEM 4 (S7, 3 POINTS)

Define a relation \sim on \mathbb{N} by setting $x \sim y$ when $x - y$ is a multiple of 5. That is, $x \sim y$ when there exists some $k \in \mathbb{Z}$ such that $x - y = 5k$. (Note that we need $k \in \mathbb{Z}$ since $x - y$ could be negative.) Is \sim an equivalence relation? If so, prove that it is. If not, explain why not.