

**2001 DISCRETE MATHEMATICS  
HOMEWORK 4  
DUE 2025 MARCH 8**

PROBLEM 1 (S4, 3 POINTS)

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = 2^x$ . Recall that the  $n^{\text{th}}$  derivative of  $f$  is  $f^{(n)}: \mathbb{R} \rightarrow \mathbb{R}$  where  $f^{(0)}(x) = f(x)$  and  $f^{(n+1)}(x) = \frac{d}{dx}f^{(n)}(x)$  for  $n \in \mathbb{N}$ . In this case,  $f^{(0)}(x) = f(x) = 2^x$  and

$$f^{(1)}(x) = \frac{d}{dx}2^x = \log(2) \cdot 2^x,$$

where  $\log$  denotes the natural logarithm with base  $e$ .

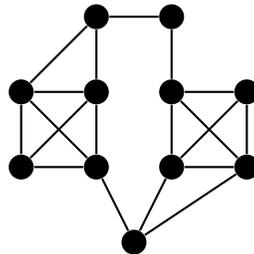
Use a proof by induction to show that  $f^{(n)}(x) = (\log(2))^n 2^x$ .

PROBLEM 2 (S5, 2 POINTS)

Suppose that a graph has 6 vertices, which have degrees 4, 3, 3, 3, 3, and 2. How many edges must the graph have?

PROBLEM 3 (S5, 2 POINTS)

Find a spanning tree for the following graph:



PROBLEM 4 (S5, 3 POINTS)

Determine whether the following graph is planar. If it is planar, give a planar drawing of it. If it is non-planar, explain how you know that.

