2001 DISCRETE MATHEMATICS
HOMEWORK 3
DUE 2025 FEBRUARY 22

Problem 1 (S3, 2 points)

Use the first-order analogue of De Morgan's Laws to negate the following sentences. (You do not need to determine whether the statements are true.)

a. There exists a prime number whose square is prime.

b. All ravens are black.

Problem 2 (S3, 2 points)

For each of the following logical equivalences, use a truth table to demonstrate the equivalence. Show intermediate steps, not just the final result.

a. $P \to (P \wedge Q) \equiv \neg(P \wedge \neg Q)$

b. $(P \to Q) \wedge (Q \to P) \equiv (P \vee Q) \to (P \wedge Q)$

Problem 3 (S4, 3 points)

In this problem and the following one, we say that a number $n \in \mathbb{N}$ is *threeven* when $n = 3k$ for some $k \in \mathbb{N}$ and *throdd* when $n = 3k + 1$ for some $k \in \mathbb{N}$. For example, $15 = 3(5)$ is threeven and $16 = 3(5) + 1$ is throdd. The number 17 is neither threeven nor throdd. You may use the fact that a number cannot be both threeven and throdd without proving it, if you wish.

Prove that if n is throdd then n^2 is also throdd.

Problem 4 (S4, 3 points)

Prove that if n is not threeven then n cannot be the sum of three throdd numbers.