MATH 2001 DISCRETE MATHEMATICS
HOMEWORK 1
DUE 2025 JANUARY 25

Problem 1 (S1, 2 points)

Find the least element of the following sets. (Show some work to explain your answer.)

a. $\{ n \in \mathbb{N} \mid 2^n \geq 10 \}$

b. $\left\{ n^{n+1} \;\middle|\; n \in \mathbb{N} \right\}$

c. $\left\{ n \in \mathbb{N} \;\middle|\; n = k^3 - 1 \text{ for some } k \in \mathbb{N} \right\}$

d. $\left\{ n \in \mathbb{N} \;\middle|\; \sqrt{n+3} \in \mathbb{N} \right\}$

Problem 2 (S1, 2 points)

Compute the following cardinalities. (Show some work to explain your answer.)

a. $|A|$ when $A = \{10, 20, 30, \ldots, 90\}$

b. $|A|$ when $A = \{ x \in \mathbb{Z} \mid -2 \leq x \leq 27 \}$

c. $|A \cap B|$ when $A = \{ n \in \mathbb{N} \mid n \leq 27 \}$ and $B = \{ n^n \in \mathbb{N} \mid n \in \mathbb{N} \}$
(Recall that $0^0 = 1$.)

d. $|A \cup B|$ when $A = \{ x \in \mathbb{Z} \mid -3 \leq x \leq 4 \}$ and $B = \{ y \in \mathbb{Z} \mid -2 \leq y \leq 6 \}$

Problem 3 (S1, 2 points)

Let $A = \{1, 2, 3, 4, 5\}$. List all the elements of

$$\{ S \in \mathcal{P}(A) \mid 2 \in S \text{ and } |S| = 3 \}.$$

Problem 4 (S1, 4 points)

Let $A = \{ n \in \mathbb{N} \mid 4 \leq n \leq 14 \}$ and $B = \left\{ n \in \mathbb{N} \;\middle|\; n^2 \leq 40 \right\}$.

a. Find $A \cap B$.

b. Find $A \cup B$.

c. Find $A \setminus B$.

d. Find $B \setminus A$.