Math 2001
Discrete Mathematics
Week 2
Set operations

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Today's topics

1. Set operations

Set operations

- To remind you of what we talked about last time, I will use Venn diagrams to show the *absorption laws* hold for unions and intersections:

$$(A \cup B) \cap A = A \text{ and } (A \cap B) \cup A = A$$

Set operations

- We have a set operation kind of like subtraction.
- The *difference* of A and B consists of all members of A which do not belong to B:

$$A \setminus B = \{\, x \in A \mid x \notin B \,\}\,.$$

- For example,

$$\{a, b\} \setminus \{a, c\} = \{b\}\,.$$

Set operations

- Note that $A \setminus B$ still makes sense when B is not a subset of A.
- We don't typically have that $|A \setminus B| = |A| - |B|$, but this is the case if $B \subset A$.
- If all sets under consideration are taken to be subsets of a big set S (like the box we might put around a Venn diagram), we can consider all elements **not** in a particular set A. This is called the *complement* of A (in S):

$$\overline{A} = \{ x \in S \mid x \notin A \}$$

- That is, $|A|$ is shorthand for $S \setminus A$ in this situation.

Set operations

- While $|A \cup B| = |A| + |B|$ when $A \cap B = \varnothing$, this is not usually the case.
- Since $|A \cap B| \leq |A|$ it is even rarer that $|A \cap B| = |A| \cdot |B|$.
- Another pair of operations does do this, however.

Set operations

- The *disjoint union* of A and B consists of all elements of A («tagged» to indicate they came from A) as well as all elements of B («tagged» to indicate they came from B):

$$A \uplus B = \{ x_A \mid x \in A \} \cup \{ y_B \mid y \in B \} .$$

- Sometimes we write $A + B$ instead of $A \uplus B$.
- The *product* of A and B consists of all ordered pairs (x, y) where $x \in A$ and $y \in B$:

$$A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$$

Set operations

- Note that $|A \uplus B| = |A| + |B|$ for all sets A and B while $|A \times B| = |A| \cdot |B|$ for all sets A and B.
- These operations are also associative, but the product is not commutative.
- Note that $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ in this sense.