Math 2001
Discrete Mathematics
Week 1
The powerset and set operations

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Today's topics

1. The power set
2. Set operations

Subsets

- As a reminder of the discussion of subsets from last time, I'll draw a diagram showing all of the subsets of a set with three elements.

The powerset

- Given a set S, the set whose members are the subsets of S is called the *power set* of S and is written $\mathcal{P}(S)$.
- For example,

$$\mathcal{P}(\{a, b\}) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$$

The powerset

- Note that $|\varnothing| = 0$ while $|\mathcal{P}(\varnothing)| = |\{\varnothing\}| = 1$.
- In general we have that if $|S| = n$ then $|\mathcal{P}(S)| = 2^n$.
- This is because we must choose either $x \in A$ or $x \notin A$ for each $x \in S$ in order to make a subset A of S.

Set operations

- There are (at least) two natural ways to make a new set from two given sets.
- The *union* of A and B consists of all elements which belong to either A or B (or both):

$$A \cup B = \{\, x \mid x \in A \text{ or } x \in B \,\}$$

- The *intersection* of A and B consists of all elements which belong to both A and B at the same time:

$$A \cap B = \{\, x \mid x \in A \text{ and } x \in B \,\}$$

Set operations

- For example,
$$\{a, b\} \cup \{a, c\} = \{a, b, c\}$$
and
$$\{a, b\} \cap \{a, c\} = \{a\}.$$
- These examples show that $|A \cup B| \neq |A| + |B|$ and $|A \cap B| \neq |A| \cdot |B|$ in general, but we can think of these operations as something like addition and multiplication for sets.

Set operations

- Union and intersection are very closely related to «(inclusive) or» and «and». We'll discuss the connection to Boolean logic a bit later in the course.
- We can visualize the operations with Venn diagrams.
- These diagrams can be useful for establishing arithmetic properties of union and intersection.

Set operations

- Just like addition and multiplication, intersection and union are:
- Associative
$$A \cup (B \cup C) = (A \cup B) \cup C$$
and
$$A \cap (B \cap C) = (A \cap B) \cap C$$
- Commutative
$$A \cup B = B \cup A \text{ and } A \cap B = B \cap A$$

Set operations

- Unlike addition and multiplication, intersection and union are:
- Idempotent

$$A \cup A = A \text{ and } A \cap A = A$$

Set operations

- Intersections distribute over unions like multiplication distributes over addition:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Unions also distribute over intersections (think $x + (yz) = (x + y)(x + z)$, which isn't true in general for numbers):

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$