Math 2001
Discrete Mathematics
Week 1
Subsets

Charlotte Aten

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Today's topics

1. Set-builder notation
2. Subsets
3. The power set

Set-builder notation

- We could write
$$\{0, 2, 4, 6, \dots\}$$
to mean the set of all (nonnegative) even numbers, but a more precise way to say this would be "All natural numbers of the form $2k$ where k can be any natural number.".
- In *set-builder notation* this is
$$\{\, 2k \in \mathbb{N} \mid k \in \mathbb{N} \,\} \,.$$
- We also sometimes write
$$\{\, 2k \mid k \in \mathbb{N} \,\}$$
when the context is clear.

Set-builder notation

- We are allowed to use natural language for the condition when defining a set using this notation.
- For example,

$$\{\, n \in \mathbb{N} \mid n \text{ is even} \,\}$$

  is a perfectly cromulent definition, assuming our readers know what an even number is.
- We could also say

$$\{\, n \in \mathbb{N} \mid n = 2k \text{ for some natural number } k \,\}.$$

Set-builder notation

- This notation allows us to specify intervals:

$$[2,9] = \{\, x \in \mathbb{R} \mid 2 \leq x \leq 9 \,\}$$

and

$$[1,5) = \{\, x \in \mathbb{R} \mid 1 \leq x < 5 \,\}.$$

Subsets

- Let $A = \left\{1, 3, \sqrt{2}\right\}$.
- It's not true that $\{1, 3\} \in A$, but every member of $\{1, 3\}$ is a member of A.
- Given sets B and A, we say that B is a *subset* of A when every member of B is also a member of A.
- We write $B \subset A$ to indicate this situation.
- It is true that $\{1, 3\} \subset A$.

Subsets

- We can think of $\subset$ like $\leq$ but for sets. You can write $\subseteq$ if you prefer.
- It is always true that $S \subset S$ for any set S.
(Think $n \leq n$ for any $n \in \mathbb{N}$.)
- It is always true that $\varnothing \subset S$ for any set S.
(Think $0 \leq n$ for any $n \in \mathbb{N}$.)
- Let's try listing all the possible subsets of some small sets.