Math 2001
Discrete Mathematics
Week 1
More about sets

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Today's topics

1. The size of sets
2. The empty set
3. Infinite sets
4. Set-builder notation

The size of sets

- We refer to the number of elements in a set as the *size* (or *cardinality*) of that set.
- Given a set A, we write $|A|$ to indicate the size of A.
- For example, $\left|\left\{1, 3, \sqrt{2}\right\}\right| = 3$.
- We won't worry right now about what $|\mathbb{Z}|$ or $|\mathbb{R}|$ mean.

The size of sets

- Note that sets are allowed to be members of other sets.
- Let $A = \{5, \{1, 3\}, \sqrt{5}\}$.
- We have that $|A| = 3$, since the elements of A are 5, $\{1, 3\}$, and $\sqrt{5}$.
- That is, the set $\{1, 3\}$ is counted as a single element of A.
- Similarly, it is not the case that $1 \in A$ even though $1 \in \{1, 3\}$.
- The membership relation cannot “look inside” the elements of A which happen to be sets.

The size of sets

- Note only do sets not care about order, as in $\left\{1, 3, \sqrt{2}\right\} = \left\{1, \sqrt{2}, 3\right\}$, we also have that sets don't care about "duplicate" entries.
- An object is either an element of a set or it is not. It can't be an element twice.
- For example, $\left\{1, 3, 3, \sqrt{2}\right\} = \left\{1, \sqrt{2}, 3\right\}$, so $\left|\left\{1, 3, 3, \sqrt{2}\right\}\right|$ is 3, not 4.

Infinite sets

- We can use ellipsis to indicate a continuing pattern when referring to an infinite set.
- The integers are

  $$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

  while the natural numbers are

  $$\mathbb{N} = \{0, 1, 2, \ldots\}.$$

- We can only do this when the context makes it clear to the reader which pattern we mean.

Infinite sets

- For instance,
$$\{3, 5, 7, \dots\}$$
could refer to the set of all odd prime numbers, or it could refer to the set of all odd numbers which are at least 3.
- Therefore, we should be careful when using this method to specify an infinite set.
- Fortunately, there is a better way in many cases.

Set-builder notation

- We could write
$$\{0, 2, 4, 6, \dots\}$$
to mean the set of all (nonnegative) even numbers, but a more precise way to say this would be "All natural numbers of the form $2k$ where k can be any natural number.".
- In *set-builder notation* this is
$$\{\, 2k \in \mathbb{N} \mid k \in \mathbb{N} \,\}\,.$$
- We also sometimes write
$$\{\, 2k \mid k \in \mathbb{N} \,\}$$
when the context is clear.