

MATH 2130 LINEAR ALGEBRA
WEEK 6 QUIZ
2025 OCTOBER 3

PROBLEM P1-1

For which values of k are there no solutions, many solutions, or a unique solution to the system

$$5x - 2y = k$$

and

$$20x - 8y = k + 2?$$

PROBLEM P1-2

Use Gauss's method to find the unique solution to the system

$$x + 2y - z = 2,$$

$$x + 4y - 2z = 0,$$

and

$$-x - y + 2z = 1.$$

PROBLEM P2-1

Find the reduced echelon form of the matrix

$$\begin{bmatrix} 7 & 3 & 2 & 2 \\ 2 & -4 & 0 & 0 \\ 1 & 15 & 2 & 2 \end{bmatrix}.$$

PROBLEM P2-2

Use Gauss-Jordan reduction to solve the system

$$x_1 + x_2 + x_3 = -2,$$

$$x_1 + x_2 + 3x_3 = -2,$$

and

$$4x_1 + 2x_2 + x_3 = -2.$$

PROBLEM P3-1

Show that

$$\{ (x, y, z) \in \mathbb{R}^3 \mid 4x + 3y + z = 0 \}$$

is closed under addition.

PROBLEM P3-2

Show that

$$\left\{ \begin{bmatrix} a & a+b \\ b & a+2 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

is not a vector space under the usual matrix operations.

PROBLEM P4-1

Show that $\{(1, 1, 6), (2, 0, 1), (1, 1, -1)\}$ is a basis for $\mathbb{R}^3$.

PROBLEM P4-2

Show that $\{x^2 + x + 1, x + 1, 4x\}$ is a basis for $\mathcal{P}_2$.

PROBLEM S1

Describe the set of points on the plane through $(1, -2, 0, -4)$, $(1, 1, 1, 1)$, and $(2, 4, 1, 0)$ in $\mathbb{R}^4$. Does the origin lie on this plane? (Use the method we saw in class. Even if you know the cross product method, it won't work in $\mathbb{R}^4$ anyway.)

PROBLEM S2

Find the angle between the vectors $(1, 0, 2, 5)$ and $(1, -1, 1, -2)$ in $\mathbb{R}^4$.

PROBLEM S3

Use the Subspace Test to show that

$$V = \{ (x, y, z) \in \mathbb{R}^3 \mid 3x - y + 2z = 0 \}$$

is a subspace of $\mathbb{R}^3$.

PROBLEM S4

Show that $\{(1, 1, 0), (1, 2, 0), (2, 3, 0), (1, 1, 1)\}$ is a spanning set for $\mathbb{R}^3$.

PROBLEM S5

Show that $\{(6, 8, 2), (1, 2, 3), (11, 14, 1)\}$ is linearly dependent in $\mathbb{R}^3$.