

MATH 2130 LINEAR ALGEBRA
WEEK 12 QUIZ
2025 NOVEMBER 14

PROBLEM P1-1

For which values of k are there no solutions, many solutions, or a unique solution to the system

$$2x + 3y = k$$

and

$$-6x - 9y = k + 4?$$

PROBLEM P1-2

Use Gauss's method to find the unique solution to the system

$$x + 2y + 2z = 3,$$

$$x + 5z = 2,$$

and

$$x + y + 3z = 3.$$

PROBLEM P2-1

Find the reduced echelon form of the matrix

$$\begin{bmatrix} 5 & 3 & 1 & 0 \\ 2 & 3 & 4 & 6 \\ 1 & 1 & 2 & 4 \end{bmatrix}.$$

PROBLEM P2-2

Use Gauss-Jordan reduction to solve the system

$$x_1 + 4x_2 + 8x_3 = 6,$$

$$2x_1 + x_2 - 3x_3 = -2,$$

and

$$x_1 + 2x_2 + x_3 = 2.$$

PROBLEM P3-1

Show that

$$\{ (x, y, z, w) \in \mathbb{R}^4 \mid 4x - 2y + z - 3w = 0 \}$$

is closed under addition.

PROBLEM P3-2

Show that

$$\{ (x, y, x^3) \in \mathbb{R}^3 \mid x, y \in \mathbb{R} \}$$

is not a vector space under the usual vector operations.

PROBLEM P4-1

Show that $\{(1, 2, 3), (1, 0, 1), (3, 2, 1)\}$ is a basis for $\mathbb{R}^3$.

PROBLEM P4-2

Show that $\{9x^2 - 2x + 1, 5x^2 + x, 4x^2 - x + 1\}$ is a basis for $\mathcal{P}_2$.

PROBLEM P5-1

Show that the function $f: \mathbb{R}^3 \rightarrow \mathcal{P}_2$ given by

$$f(a, b, c) = (a + b)x^2 + (a + c)x + (a + b + c)$$

is a homomorphism.

PROBLEM P5-2

Is the function $g: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $g(a, b, c) = (a + b, a - 2b + c, 2a + 2b)$ an isomorphism? Either prove that it is or show that one of the conditions fails.

PROBLEM P6-1

Let $f: \mathcal{P}_2 \rightarrow \text{Mat}_{2 \times 2}$ be given by

$$f(ax^2 + bx + c) = \begin{bmatrix} a + 2b & c \\ a + 2b - 3c & c \end{bmatrix}.$$

Find a basis for $\text{Ker}(f)$.

PROBLEM P6-2

Let $g: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by

$$g(x, y, z) = (x + y, x + z, y - z).$$

Find a basis for $\text{Im}(g)$.

PROBLEM S1

Describe the set of points on the plane through $(1, 0, 3, 1)$, $(0, 1, 2, 2)$, and $(2, 4, 1, 3)$ in $\mathbb{R}^4$. Does the origin lie on this plane?

PROBLEM S2

Find the angle between the vectors $(1, 0, 3, 1)$ and $(0, 1, 2, 2)$ in $\mathbb{R}^4$.

PROBLEM S3

Use the Subspace Test to show that

$$V = \{ ax^2 + bx + c \in \mathcal{P}_2 \mid a + 2b - c = 0 \}$$

is a subspace of $\mathcal{P}_2$.

PROBLEM S4

Show that $\{x^2 + x + 1, x^2 + 1, x + 1, 3x^2\}$ is a spanning set for $\mathcal{P}_2$.

PROBLEM S5

Show that $\{(1, 2, 3), (-1, 0, 1), (2, 6, 10)\}$ is linearly dependent in $\mathbb{R}^3$.

PROBLEM S6

Evaluate

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 & -1 \\ 3 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}.$$

PROBLEM S7

Find the inverse matrix of

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}.$$

PROBLEM S8

Let $h: \mathcal{P}_2 \rightarrow \mathbb{R}^2$ be given by $h(ax^2 + bx + c) = (a + c, b + c)$ and let $B = \{1, x, 3x^2 - 1\}$ and $C = \{(1, 1), (2, 0)\}$ be bases for $\mathcal{P}_2$ and $\mathbb{R}^2$, respectively. Find $[h]_B^C$.