

MATH 2130 LINEAR ALGEBRA
HOMEWORK 7
DUE 2025 OCTOBER 12

PROBLEM 1 (P4)

Show that $\{(1, 6, 0), (2, 3, -1), (2, 2, 2)\}$ is a basis for $\mathbb{R}^3$.

PROBLEM 2 (P5)

Is the function $f: \mathbb{R}^3 \rightarrow \mathcal{P}_2$ given by

$$f(a, b, c) = ax^2 + (a - 2b)x + (a + c)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.

PROBLEM 3 (P5)

Is the function $g: \mathbb{R}^2 \rightarrow \mathcal{P}_1$ given by

$$g(a, b) = (a + b)x + (a - b)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.

PROBLEM 4 (P5)

Is the function $h: \text{Mat}_{2 \times 2} \rightarrow \mathbb{R}^4$ given by

$$h \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a + b, a - 2c + d, 2c + d, 2a + b + 2d)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.

PROBLEM 5 (P5)

Is the function $\psi: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $\psi(a, b) = (a + b, a + b^2)$ an isomorphism? Either prove that it is or show that one of the conditions fails.