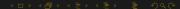
# Math 2130 Linear Algebra Week 8 Matrix multiplication

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# Today's topics

- Let *A*, *B*, and *C* be matrices of the appropriate sizes so that the indicated operations may be performed. All of the following are true:
- A(BC) = (AB)C (matrix multiplication is associative)
- A(B+C) = AB + AC (matrix multiplication on the left distributes over addition)
- (A + B)C = AC + BC (matrix multiplication on the right distributes over addition)

- We use the same notation for multiplying a matrix by itself as we do for multiplying a scalar by itself.
- That is,

$$A^2 = AA, A^3 = AAA, A^4 = AAAA,$$

and so forth.

Given that

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

compute  $A^3$ .

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Since  $A^3=A^2A$  and

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

we have that

$$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

#### **Definition**

The  $n \times n$  matrix

$$I_n = [\delta_{ij}]$$

where

$$\delta_{ij} = egin{cases} 1 & ext{when } i=j \ 0 & ext{otherwise} \end{cases}$$

is called the *identity matrix* of size  $n \times n$ .

We have that

$$I_1=\begin{bmatrix}1\end{bmatrix}$$
 ,  $I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$  , and  $I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$  .

- The identity matrix plays the same role in matrix multiplication that 1 does in the usual multiplication.
- $\blacksquare$  If A is any  $m \times n$  matrix then

$$I_m A = A = A I_n.$$

- Note that if we identify 1 with  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $i = \sqrt{-1}$  with  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  then we can think of complex numbers as matrices.
- That is, a+bi can be identified with  $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$  and addition and multiplication of complex numbers is compatible with this identification.

In the usual multiplication we have that 1 has only two square roots, which are 1 and -1.

Find all matrices A of the form

$$A = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$$

such that  $A^2 = I_2$ .

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Find all matrices A of the form

$$A = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$$

such that  $A^2 = I_2$ . We have

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 = A^2 = \begin{bmatrix} a^2 - 1 & a + b \\ -a - b & -1 + b^2 \end{bmatrix}$$

so b = -a and  $a = \pm \sqrt{2}$ .

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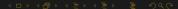
Find all matrices A of the form

$$A = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$$

such that  $A^2 = I_2$ .

We find that

$$\begin{bmatrix} \sqrt{2} & 1 \\ -1 & -\sqrt{2} \end{bmatrix}^2 = I_2 = \begin{bmatrix} -\sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}^2.$$



■ We see that the Fundamental Theorem of Algebra is not true for matrix algebra, so it is possible that a given polynomial equation like  $x^2-1=0$  might have more solutions than its degree when we interpret it as a matrix equation  $A^2-I=0$ .

- It is not the case that any finite power of a matrix needs to be the identity matrix.
- For example, consider  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ .