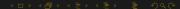
# Math 2130 Linear Algebra Week 8 Matrix multiplication

Charlotte Aten

2025 October 15



# Today's topics

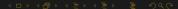
Matrix multiplication

- Let  $f: \mathbb{R}^2 \to \mathbb{R}^2$  be given by f(x,y) = (-y,x) and let  $g: \mathbb{R}^2 \to \mathbb{R}^2$  be given by g(x,y) = (2x,y).
- Using the standard basis  $B = \{(1,0),(0,1)\}$ , we can represent f as

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

and we can represent q as

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$



■ Note that  $g \circ f : \mathbb{R}^2 \to \mathbb{R}^2$  is given by

$$(g \circ f)(x, y) = g(f(x, y)) = g(-y, x) = (-2y, x),$$

which has matrix representation

$$\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}.$$

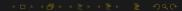
#### Definition

Given an  $m \times n$  matrix A and an  $n \times k$  matrix B where  $a_i$  is the  $i^{\text{th}}$  row of A and  $b_j$  is the  $j^{\text{th}}$  column of B the product of A and B is the  $m \times k$  matrix

$$AB = [a_i \cdot b_j].$$

■ For example, if 
$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$
 and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 4 & 7 \end{bmatrix}$  then

$$AB = \begin{bmatrix} 14 & 26 \end{bmatrix}$$
.



- We have that  $(g \circ f)(x) = (-2y, x)$  and  $(f \circ g)(x, y) = (-y, 2x)$ .
- Similarly, we have that

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

while

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}.$$

■ Typically, we don't have that AB = BA for matrices.

When  $A=[a_{ij}]$  is an  $m\times n$  matrix,  $B=[b_{ij}]$  is an  $n\times k$  matrix, and  $AB=C=[c_{ij}]$  we have that

$$c_{ij} = \sum_{\ell=1}^{n} a_{i\ell} b_{\ell j}.$$

- This is called the index form of the matrix product.
- Observe that

$$\sum_{\ell=1}^{n} a_{i\ell} b_{\ell j} = a_i \cdot b_j,$$

so this agrees with the formula we already gave.

