Math 2130
Linear Algebra
Week 7
Isomorphisms

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Today's topics

1. Isomorphisms
2. Dimension and isomorphism

Isomorphisms

Definition

Given vector spaces V and W we say that a function $f\colon V \to W$ is an *isomorphism* between V and W when

1. f is a bijection,
2. for all $v_1, v_2 \in V$ we have $f(v_1 + v_2) = f(v_1) + f(v_2)$, and
3. for all $s \in \mathbb{R}$ and all $v \in V$ we have $f(sv) = sf(v)$.

Isomorphisms

Is the function $f: \mathbb{R}^2 \to \mathcal{P}_1$ given by

$$f(a, b) = (3a + 3b)x + (3a + b)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.

Isomorphisms

Is the function $k\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$k(x, y) = (x, y^3)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.