

Math 2130
Linear Algebra
Week 7
Examples of isomorphisms

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Today's topics

1 Isomorphisms

Isomorphisms

Definition

Given vector spaces V and W we say that a function $f: V \rightarrow W$ is an *isomorphism* between V and W when

- 1 f is a bijection,
- 2 for all $v_1, v_2 \in V$ we have $f(v_1 + v_2) = f(v_1) + f(v_2)$, and
- 3 for all $s \in \mathbb{R}$ and all $v \in V$ we have $f(sv) = sf(v)$.

Isomorphisms

Definition

An *automorphism* is an isomorphism between a vector space and itself.

Isomorphism

Lemma

The inverse of an isomorphism is an isomorphism.

Isomorphisms

Lemma

For any vector spaces V and W and any function $f: V \rightarrow W$ the following are equivalent:

- 1** $f(v_1 + v_2) = f(v_1) + f(v_2)$ *and* $f(cv) = cf(v)$
- 2** $f(c_1v_1 + c_2v_2) = c_1f(v_1) + c_2f(v_2)$
- 3** $f(c_1v_1 + \cdots + c_nv_n) = c_1f(v_1) + \cdots + c_nf(v_n)$

Isomorphisms

Is the function $f: \mathbb{R}^2 \rightarrow \mathcal{P}_1$ given by

$$f(a, b) = (3a + 3b)x + (3a + b)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.