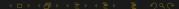
# Math 2130 Linear Algebra Week 7 Examples of isomorphisms

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# Today's topics

Isomorphisms

### Definition

Given vector spaces V and W we say that a function  $f \colon V \to W$  is an isomorphism between V and W when

- f is a bijection,
- **2** for all  $v_1, v_2 \in V$  we have  $f(v_1 + v_2) = f(v_1) + f(v_2)$ , and
- for all  $s \in \mathbb{R}$  and all  $v \in V$  we have f(sv) = sf(v).

### Definition

An *automorphism* is an isomorphism between a vector space and itself.

### Lemma

The inverse of an isomorphism is an isomorphism.

### Lemma

For any vector spaces V and W and any function  $f\colon V\to W$  the following are equivalent:

- $f(v_1 + v_2) = f(v_1) + f(v_2)$  and f(cv) = cf(v)
- $\overline{s} f(c_1v_1 + \dots + c_nv_n) = c_1f(v_1) + \dots + c_nf(v_n)$

Is the function  $f: \mathbb{R}^2 \to \mathcal{P}_1$  given by

$$f(a, b) = (3a + 3b)x + (3a + b)$$

an isomorphism? Either prove that it is or show that one of the conditions fails.