

Math 2130
Linear Algebra
Week 6
Dimension

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2025 October 1

Today's topics

- 1 Bases
- 2 Dimension

Bases

Definition (Basis)

A set of vectors $\{v_1, v_2, \dots, v_k\}$ in a vector space V is a *basis* for V when

- 1 $\text{Span}(\{v_1, v_2, \dots, v_k\}) = V$ and
- 2 $\{v_1, v_2, \dots, v_k\}$ is linearly independent.

Bases

Show that

$$\{x^2 + x + 1, x^2 + x + 2, x^2 + 2x + 3\}$$

is a basis for P_2 .

Dimension

Definition (Dimension)

The *dimension* of a vector space V is the size of any basis for V .

Dimension

The preceding definition makes sense because of the following theorem.

Theorem

Each basis for a vector space V contains the same number of vectors as any other basis for V .

Dimension

Definition (Row space)

The *row space* of an $m \times n$ matrix A is the subspace of $\mathbb{R}^n$ spanned by the rows of A . We denote the row space of A by $\text{Row}(A)$.

Dimension

- The *rank* of a matrix A is $\dim(\text{Row}(A))$.
- This is the same as the number of leading 1s in any echelon form of A .