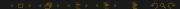
# Math 2130 Linear Algebra Week 5 Linear independence

Charlotte Aten

2025 September 26



# Today's topics

- Linear independence
- $oxed{2}$  Other vector spaces beside  $\mathbb{R}^n$

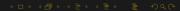
#### Definition

A finite nonempty set of vectors  $\{v_1, v_2, \ldots, v_k\}$  in a vector space V is said to be *linearly dependent* when there exist scalars  $c_1, c_2, \ldots, c_k$ , at least one of which is nonzero, such that

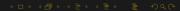
$$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = 0.$$

#### Definition

A finite nonempty set of vectors  $\{v_1, v_2, \dots, v_k\}$  in a vector space V is said to be *linearly independent* when it is not linearly dependent.



- In  $\mathbb{R}^2$  the spanning sets  $\{(1,0),(0,1)\}$  and  $\{(1,0),(1,1)\}$  are linearly independent. The spanning set  $\{(1,0),(0,1),(1,2)\}$  is linearly dependent.
- The spanning set  $\{(1,1,1),(1,1,2),(1,2,3)\}$  for  $\mathbb{R}^3$  is also linearly independent.
- Even sets which do not span can be linearly independent. For example,  $\{(1,1,1),(1,1,2)\}$  is linearly independent in  $\mathbb{R}^3$  but does not span.
- $\blacksquare$  Note that any set consisting of a single nonzero vector  $\{v\}$  is linearly independent.
- On the other hand, any set containing the zero vector is linearly dependent.



Determine whether  $\{(1,4,7),(2,5,8),(3,6,9)\}$  is linearly independent in  $\mathbb{R}^3$ .

Determine whether  $\{(1,4,7),(2,5,8),(3,6,9)\}$  is linearly independent in  $\mathbb{R}^3$ . If we have

$$c_1(1,4,7) + c_2(2,5,8) + c_3(3,6,9) = (0,0,0)$$

then we must have a homogeneous system with augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{bmatrix}.$$

Determine whether  $\{(1,4,7),(2,5,8),(3,6,9)\}$  is linearly independent in  $\mathbb{R}^3$ .

This matrix has row-echelon form

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so we can see that there is a free variable and hence infinitely many solutions. Thus, the set  $\{(1,4,7),(2,5,8),(3,6,9)\}$  is linearly dependent.

## Other vector spaces beside $\mathbb{R}^n$

- We can talk about spanning sets and linear independence for vector spaces other than  $\mathbb{R}^n$ .
- For example,  $\{2x+1,5,x^2+x+1\}$  is a spanning set for  $\mathcal{P}_2$ .