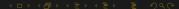
Math 2130 Linear Algebra Week 5 Spanning sets and linear independence

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Today's topics

- Spanning sets
- Linear independence

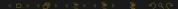
Spanning sets

Definition

If every vector in a vector space V can be written as a linear combination of $\{v_1,v_2,\ldots,v_k\}$ we say that V is spanned (or generated) by $\{v_1,v_2,\ldots,v_k\}$. The set of vectors $\{v_1,v_2,\ldots,v_k\}$ is called a spanning set for V. We also say that $\{v_1,v_2,\ldots,v_k\}$ spans V and write $\mathrm{Span}(\{v_1,v_2,\ldots,v_k\})=V$ in this situation.

lacksquare More generally, we define $\mathrm{Span}(\{v_1,v_2,\ldots,v_k\})$

$$\{c_1v_1+c_2v_2+\cdots+c_kv_k \mid c_1,c_2,\ldots,c_k \in \mathbb{R}\}.$$



Spanning sets

Theorem

Given a set of vectors $\{v_1, v_2, \dots, v_k\}$ where v_k is a linear combination of $\{v_1, v_2, \dots, v_{k-1}\}$ we have that

$$Span(\{v_1, v_2, \dots, v_k\}) = Span(\{v_1, v_2, \dots, v_{k-1}\}).$$

Spanning sets

■ Both $\{(1,0),(0,1)\}$ and $\{(1,0),(1,1)\}$ are spanning sets for \mathbb{R}^2 . By similar reasoning, or by the previous theorem, so is $\{(1,0),(0,1),(1,2)\}$.

Linear independence

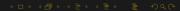
Definition

A finite nonempty set of vectors $\{v_1, v_2, \ldots, v_k\}$ in a vector space V is said to be *linearly dependent* when there exist scalars c_1, c_2, \ldots, c_k , at least one of which is nonzero, such that

$$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = 0.$$

Definition

A finite nonempty set of vectors $\{v_1, v_2, \dots, v_k\}$ in a vector space V is said to be *linearly independent* when it is not linearly dependent.



Linear independence

Theorem

A finite nonempty set of vectors $\{v_1, v_2, \ldots, v_k\}$ in a vector space V is linearly independent when the only choice of scalars c_1, c_2, \ldots, c_k so that

$$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = 0$$

is
$$c_1 = c_2 = \cdots = c_k = 0$$
.

Linear independence

■ In \mathbb{R}^2 the spanning sets $\{(1,0),(0,1)\}$ and $\{(1,0),(1,1)\}$ are linearly independent. The spanning set $\{(1,0),(0,1),(1,2)\}$ is linearly dependent.