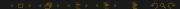
Math 2130 Linear Algebra Week 4 Vector spaces

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# Today's topics

- Vector spaces
- 2 Subspaces

#### Definition (Vector space)

Let V be a set (whose members are called  $\emph{vectors}$ ) on which addition and scalar multiplication are defined. We say that V is a  $\emph{vector space}$  when

- closure under addition) for each pair of vectors u and v in V we have that  $u+v\in V$ ,
- closure under scalar multiplication) for each vector  $v \in V$  and each scalar  $s \in \mathbb{R}$  we have that  $sv \in V$ ,
- $\blacksquare$  (commutativity of addition) for all  $u,v\in V$  we have that u+v=v+u,

#### Definition (Vector space)

- associativity of addition) for all  $u, v, w \in V$  we have that u + (v + w) = (u + v) + w,
- (zero vector) there is a vector  $0 \in V$  such that for any  $v \in V$  we have v + 0 = v,
- (additive inverse) for each  $v \in V$  there is a vector  $-v \in V$  such that v + (-v) = 0,
- (unit property) for each  $v \in V$  we have 1v = v,
- (associativity of scalar multiplication) for all  $v \in V$  and all  $r, s \in \mathbb{R}$  we have (rs)v = r(sv),

#### Definition (Vector space)

- (distributivity of scalar multiplication over vector addition) for all  $u,v\in V$  and all  $r\in \mathbb{R}$  we have that r(u+v)=ru+rv, and
- (distributivity of scalar multiplication over scalar addition) for all  $v \in V$  and all  $r, s \in \mathbb{R}$  we have that (r+s)v = rv + sv.

Show that

$$\{(x,y) \in \mathbb{R}^2 \mid (x,y) \cdot (3,-1) = 1\}$$

is not a vector space under the usual vector operations.

Show that

$$\left\{ \begin{bmatrix} a & a+b \\ b & b \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$

is closed under scalar multiplication.

Show that scalar multiplication distributes over addition for real-valued functions  $f: \mathbb{R} \to \mathbb{R}$ .

Consider the set of vectors  $\mathbb{R}^2$  where we define addition by

$$(x_1, x_2) \oplus (y_1, y_2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

and scalar multiplication is defined as usual. Show that this is not a vector space.

#### Subspaces

#### Definition

Given a vector space V and a nonempty subset S of V we say that S is a *subspace* of V when S is a vector space under the same addition and scalar multiplication used in V.

## Subspaces

■ Fortunately, we don't have to check all ten axioms discussed previously in order to see if S is a really a subspace of V.

#### Theorem (Subspace test)

A nonempty subset S of a vector space V is a subspace of V if and only if S is closed under the addition and scalar multiplication operations of V.