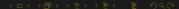
Math 2130 Linear Algebra Week 3 Vector spaces

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2025 September 12



Today's topics

Vector spaces

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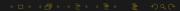
Definition

Given variables/numbers/matrices u_1, \ldots, u_n and scalars $c_1, \ldots, c_n \in \mathbb{R}$ we say that

$$c_1u_1+c_2u_2+\cdots+c_nu_n$$

is a linear combination of u_1, \ldots, u_n .

■ A system of linear equations is asking us to solve something like $xu_1 + yu_2 + zu_3 = w$ where u_1 , u_2 , u_3 , and w are vectors.



Vector spaces

- We have seen many different collections of objects, such as vectors in \mathbb{R}^n and various sized matrices, which can be added as well as multiplied by scalars.
- At this point it may seem reasonable that the arithmetic of matrices should be thought of as some kind of generalization of the arithmetic of real numbers themselves.
- The concept of a vector space encompasses this idea, as well as several others we will use throughout the rest of the course.

Examples of vector spaces

- The vectors in \mathbb{R}^n where $(x_1,\ldots,x_n)+(y_1,\ldots,y_n)=(x_1+y_1+,\ldots,x_n+y_n)$ and $s(x_1,\ldots,x_n)=(sx_1,\ldots,sx_n).$
- The numbers in $\mathbb R$ with the usual addition and multiplication. Note that $\mathbb R = \mathbb R^1.$
- The matrices $\mathrm{Mat}_{m \times n}$ of size $m \times n$ where addition and scalar multiplication are done componentwise.

Examples of vector spaces

- The set of vectors $\{(x,y) \mid 2x+3y=0\}$ with the usual addition and scalar multiplication from \mathbb{R}^2 .
- The set of vectors $\{(x,y,z) \mid (x,y,z) \cdot (1,-1,4) = 0\}$ with the usual addition and scalar multiplication from \mathbb{R}^3 .

Examples of vector spaces

■ The set of all functions $f: \mathbb{R} \to \mathbb{R}$ where (f+g)(x) = f(x) + g(x) and (sf)(x) = s(f(x)).