Math 2130
Linear Algebra
Week 3
Gauss-Jordan reduction

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Today's topics

1. Gauss-Jordan reduction

Gauss's method

- Small systems of linear equations may be solved by substitution, but this is difficult (or impossible) for larger systems.
- There are, however, three basic ways we can change our view of a system of linear equations which can help us find solutions:
  1. Swap two equations.
  2. Add a nonzero multiple of one equation to another one.
  3. Multiply an equation by a nonzero scalar.

Matrices and linear systems

Definition

A $m \times n$ matrix is said to be a *row-echelon matrix* when

1. all rows consisting entirely of zeroes are at the bottom of the matrix,
2. the first nonzero entry in any nonzero row is a 1 (called the *leading* 1), and
3. the leading 1 of any row below the first row is to the right of the leading 1 of the row above it.

Gauss-Jordan reduction

Definition

We say that a matrix (or linear system) is in *reduced echelon form* if each leading entry is a 1 and is the only nonzero entry in its column.

- Each matrix is equivalent to a unique reduced echelon matrix.

Gauss-Jordan reduction

Definition

Given variables/numbers/matrices $u_1, \ldots, u_n$ and scalars $c_1, \ldots, c_n \in \mathbb{R}$ we say that

$$c_1 u_1 + c_2 u_2 + \cdots + c_n u_n$$

is a *linear combination* of $u_1, \ldots, u_n$.

- Note that a linear combination of linear combinations is a linear combination.
- In an echelon form of a matrix, no nonzero row is a linear combination of the other nonzero rows.