Math 2130
Linear Algebra
Week 3
Gauss-Jordan reduction

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Today's topics

1. Gauss-Jordan reduction

Matrices and linear systems

Definition

A $m \times n$ matrix is said to be a *row-echelon matrix* when

1. all rows consisting entirely of zeroes are at the bottom of the matrix,
2. the first nonzero entry in any nonzero row is a 1 (called the *leading* 1), and
3. the leading 1 of any row below the first row is to the right of the leading 1 of the row above it.

Matrices and linear systems

- The three basic moves from Gauss's method correspond to the following *elementary row operations* on the augmented matrix $A^{\#}$ of the system:
    1. P_{ij}: Swap rows i and j.
    2. $M_i(k)$: Multiply each entry in row i by $k \neq 0$.
    3. $A_{ij}(k)$: Add k times row i to row j.
- Each of these operations can be «undone» by applying another appropriate operation.
- For example, we can reverse multiplying a row by k by multiplying that same row by $\frac{1}{k}$.

Matrices and linear systems

Definition

Given an $m \times n$ matrix A we say that any matrix obtained from A by a finite sequence of elementary row operations is *row-equivalent* to A.

Matrices and linear systems

Theorem

Systems of linear equations with row-equivalent augmented matrices have the same solution sets.

Theorem

Every matrix is row-equivalent to a row-echelon matrix.

There is an algorithm for doing this. I encourage you to practice using the basic operations and look at many examples before trying to memorize the algorithm. The process should become second-nature without needing to memorize the algorithm explicitly.

Gauss-Jordan reduction

Definition

We say that a matrix (or linear system) is in *reduced echelon form* if each leading entry is a 1 and is the only nonzero entry in its column.

- Each matrix is equivalent to a unique reduced echelon matrix.

Gauss-Jordan reduction

Definition

Given variables/numbers/matrices $u_1, \ldots, u_n$ and scalars $c_1, \ldots, c_n \in \mathbb{R}$ we say that

$$c_1 u_1 + c_2 u_2 + \cdots + c_n u_n$$

is a *linear combination* of $u_1, \ldots, u_n$.

- Note that a linear combination of linear combinations is a linear combination.
- In an echelon form of a matrix, no nonzero row is a linear combination of the other nonzero rows.