Math 2130
Linear Algebra
Week 2
Lengths and angles

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Today's topics

1. Lengths
2. Angles

Vectors

- We may think of a vector in $\mathbb{R}^n$ as consisting of a magnitude (or length) and a direction.
- Linear equations may represent lines, planes, or their higher-dimensional analogues.

Length

- The *length* of a vector $(v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$ is

$$|v| = \sqrt{v_1^2 + \cdots + v_n^2}.$$

Length

Find the length of the vector $(1, 2, 3, 4)$ in $\mathbb{R}^4$.

Angles

- The *dot product* of $u = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n$ and $v = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$ is

$$u \cdot v = u_1 v_1 + \cdots + u_n v_n.$$

- We have that $u \cdot v = |u|\,|v| \cos(\theta)$ where θ is the angle between u and v.

Angles

Find the angle between the vectors $(0, 1, 2)$ and $(1, -1, 4)$ in $\mathbb{R}^3$.

Angles

- The *dot product* of $u = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n$ and $v = (v_1, v_2, \ldots, v_n) \in \mathbb{R}^n$ is

$$u \cdot v = u_1 v_1 + \cdots + u_n v_n.$$

- We have that $u \cdot v = |u|\,|v| \cos(\theta)$ where θ is the angle between u and v.
- When $u \cdot v = 0$ (i.e. when the angle between u and v is $\frac{\pi}{2}$), we say that u and v are *orthogonal*.

Angles

For which values of k are $(-4, 2, k)$ and $(k, 2, k)$ orthogonal?