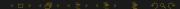
# Math 2130 Linear Algebra Week 14 Eigenvectors and eigenvalues

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# Today's topics

Eigenvectors and eigenvalues

We find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}.$$

- The characteristic polynomial of A is  $\lambda^3 2\lambda^2 3\lambda$ .
- We find that A has three real eigenvalues, -1, 0, and 3.

- Now we find the eigenvectors for A.
- Let  $v = (v_1, v_2, v_3)$ .
- Those vectors which satisfy Av = -1v have

$$0 = -v_1$$

$$v_1 + 2v_2 + 3v_3 = -v_2$$

$$2v_1 + v_2 = -v_3$$

■ Solving the system, we see that  $v_1=0$  and  $v_3=-v_2$ , so the eigenvectors for A with eigenvalue -1 are those of the form (0,x,-x) where  $x\neq 0$ .

- Let  $v = (v_1, v_2, v_3)$ .
- Those vectors which satisfy Av = 0v have

$$0 = 0$$
$$v_1 + 2v_2 + 3v_3 = 0$$
$$2v_1 + v_2 = 0$$

- Solving the system, we see that  $v_2 = -2v_1$  and  $v_3 = v_1$ , so the eigenvectors for A with eigenvalue 0 are those of the form (x, -2x, x) where  $x \neq 0$ .
- lue Note that this is just the kernel of A.

- Let  $v = (v_1, v_2, v_3)$ .
- Those vectors which satisfy Av = 3v have

$$0 = 3v_1$$
$$v_1 + 2v_2 + 3v_3 = 3v_2$$
$$2v_1 + v_2 = 3v_3$$

■ Solving the system, we see that  $v_1=0$  and  $v_2=3v_3$ , so the eigenvectors for A with eigenvalue -1 are those of the form (0,3x,x) where  $x\neq 0$ .

We find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

- The characteristic polynomial of A is  $\lambda^3 5\lambda^2 + 7\lambda 3\lambda^2 + 7\lambda = 3\lambda^2 + 7\lambda + 3\lambda^2 + 3$
- If we hadn't gotten this factored already as  $(\lambda 1)^2(\lambda 3)$  when we computed  $\det(\lambda I A)$  then we would have had to use the cubic formula or some other method to find the roots.
- $\blacksquare$  We find that A has two eigenvalues, 1 and 3.

- Now we find the eigenvectors for A.
- Let  $v = (v_1, v_2, v_3)$ .
- Those vectors which satisfy Av = 1v have

$$v_1 + 2v_2 = v_1$$
$$v_2 = v_2$$
$$3v_3 = v_3$$

■ Solving the system, we see that  $v_2 = 0$  and  $v_3 = 0$ , so the eigenvectors for A with eigenvalue 1 are those of the form (x,0,0) where  $x \neq 0$ .

- Let  $v = (v_1, v_2, v_3)$ .
- Those vectors which satisfy Av = 3v have

$$v_1 + 2v_2 = 3v_1$$
$$v_2 = 3v_2$$
$$3v_3 = 3v_3$$

■ Solving the system, we see that  $v_1 = 0$  and  $v_2 = 0$ , so the eigenvectors for A with eigenvalue 3 are those of the form (0,0,x) where  $x \neq 0$ .

■ Again we see that a matrix can have an eigenvalue with multiplicity 2 in the characteristic polynomial while the corresponding eigenvectors (along with the zero vector) only constitute a 1-dimensional space.