Math 2130
Linear Algebra
Week 14
Eigenvectors and eigenvalues

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Today's topics

1. Eigenvectors and eigenvalues

Eigenvectors and eigenvalues

- We find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}.$$

- The characteristic polynomial of A is $\lambda^3 - 2\lambda^2 - 3\lambda$.
- We find that A has three real eigenvalues, -1, 0, and 3.

Eigenvectors and eigenvalues

- Now we find the eigenvectors for A.
- Let $v = (v_1, v_2, v_3)$.
- Those vectors which satisfy $Av = -1v$ have

$$\begin{aligned} 0 &= -v_1 \\ v_1 + 2v_2 + 3v_3 &= -v_2 \\ 2v_1 + v_2 &= -v_3 \end{aligned}$$

- Solving the system, we see that $v_1 = 0$ and $v_3 = -v_2$, so the eigenvectors for A with eigenvalue -1 are those of the form $(0, x, -x)$ where $x \neq 0$.

Eigenvectors and eigenvalues

- Let $v = (v_1, v_2, v_3)$.
- Those vectors which satisfy $Av = 0v$ have

$$\begin{aligned} 0 &= 0 \\ v_1 + 2v_2 + 3v_3 &= 0 \\ 2v_1 + v_2 &= 0 \end{aligned}$$

- Solving the system, we see that $v_2 = -2v_1$ and $v_3 = v_1$, so the eigenvectors for A with eigenvalue 0 are those of the form $(x, -2x, x)$ where $x \neq 0$.
- Note that this is just the kernel of A.

Eigenvectors and eigenvalues

- Let $v = (v_1, v_2, v_3)$.
- Those vectors which satisfy $Av = 3v$ have

$$\begin{aligned} 0 &= 3v_1 \\ v_1 + 2v_2 + 3v_3 &= 3v_2 \\ 2v_1 + v_2 &= 3v_3 \end{aligned}$$

- Solving the system, we see that $v_1 = 0$ and $v_2 = 3v_3$, so the eigenvectors for A with eigenvalue -1 are those of the form $(0, 3x, x)$ where $x \neq 0$.

Eigenvectors and eigenvalues

- We find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

- The characteristic polynomial of A is $\lambda^3 - 5\lambda^2 + 7\lambda - 3$.
- If we hadn't gotten this factored already as $(\lambda - 1)^2(\lambda - 3)$ when we computed $\det(\lambda I - A)$ then we would have had to use the cubic formula or some other method to find the roots.
- We find that A has two eigenvalues, 1 and 3.

Eigenvectors and eigenvalues

- Now we find the eigenvectors for A.
- Let $v = (v_1, v_2, v_3)$.
- Those vectors which satisfy $Av = 1v$ have

$$\begin{aligned} v_1 + 2v_2 &= v_1 \\ v_2 &= v_2 \\ 3v_3 &= v_3 \end{aligned}$$

- Solving the system, we see that $v_2 = 0$ and $v_3 = 0$, so the eigenvectors for A with eigenvalue 1 are those of the form $(x, 0, 0)$ where $x \neq 0$.

Eigenvectors and eigenvalues

- Let $v = (v_1, v_2, v_3)$.
- Those vectors which satisfy $Av = 3v$ have

$$\begin{aligned} v_1 + 2v_2 &= 3v_1 \\ v_2 &= 3v_2 \\ 3v_3 &= 3v_3 \end{aligned}$$

- Solving the system, we see that $v_1 = 0$ and $v_2 = 0$, so the eigenvectors for A with eigenvalue 3 are those of the form $(0, 0, x)$ where $x \neq 0$.

Eigenvectors and eigenvalues

- Again we see that a matrix can have an eigenvalue with multiplicity 2 in the characteristic polynomial while the corresponding eigenvectors (along with the zero vector) only constitute a 1-dimensional space.