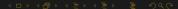
Math 2130 Linear Algebra Week 13 Determinants

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Today's topics

- Exploration
- Properties of determinants

- A 1×1 matrix [a] is nonsingular if and only if $a \neq 0$.
- lacksquare A 2 imes 2 matrix $egin{bmatrix} a & b \ c & d \end{bmatrix}$ is nonsingular if and only if ad-bc
 eq 0.
- \blacksquare One can show that a 3×3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

is nonsingular if and only if

$$aei + bfg + cdh - hfa - idb - gec \neq 0.$$



It seems that there may be, for each n, a determinant function

$$\det_n: \operatorname{Mat}_{n \times n} \to \mathbb{R}$$

where $\det_n(A) \neq 0$ exactly when A is nonsingular.

■ To find what such a formula would look like, we check how the known determinant functions behave with respect to the elementary row operations.

- Adding a multiple of one row to another doesn't change any of the potential determinant formulas we found before.
- Swapping two rows negates the previously given formulas.
- Multiplying a row by a scalar multiplies the value of the formula by the same scalar.

Properties of determinants

Definition

An $n \times n$ determinant is a function $\det: \operatorname{Mat}_{n \times n} \to \mathbb{R}$ such that

- l $\det(\rho_1,\ldots,k\rho_i+\rho_j,\ldots,\rho_n)=\det(\rho_1,\ldots,\rho_j,\ldots,\rho_n)$ when $i\neq j$,
- $\det(\rho_1, \dots, \rho_j, \dots, \rho_i, \dots, \rho_n) = \\ -\det(\rho_1, \dots, \rho_i, \dots, \rho_j, \dots, \rho_n) \text{ when } i \neq j,$
- $\det(\rho_1,\ldots,k\rho_i,\ldots,\rho_n)=k\det(\rho_1,\ldots,\rho_i,\ldots,\rho_n)$ for any scalar k, and