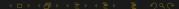
Math 2130 Linear Algebra Week 12 Changing representations

Charlotte Aten

2025 November 12



Today's topics

Changing representations

- The *identity homomorphism* for a vector space V is the function $id: V \to V$ given by id(v) = v.
- Given any basis B for V, we have that $[\mathrm{id}]_B^B = I_n$ where $n = \dim(V)$.
- We sometimes write id_V to emphasize that we are talking about the identity homomorphism for the vector space V, as opposed to the identity homomorphism for some other vector space.
- Note that if $h: V \to W$ is a homomorphism then $\mathrm{id}_W \circ h = h = h \circ \mathrm{id}_V.$
- In particular, $id_V \circ id_V = id_V$.

Proposition

Given a homomorphism $h\colon V\to W$, bases B_1 and B_2 for V, and bases C_1 and C_2 for W we have that

$$[h]_{B_1}^{C_1}[\operatorname{id}_V]_{B_2}^{B_1} = [h]_{B_2}^{C_1}$$

and

$$[\mathrm{id}_W]_{C_1}^{C_2}[h]_{B_1}^{C_1} = [h]_{B_1}^{C_2}.$$



Definition (Change of basis matrix)

Given a vector space V and two bases B and C, the *change of basis* matrix from B to C is

$$[\mathrm{id}]_B^C$$

where $id: V \to V$ is the identity homomorphism for V.

Proposition

A square matrix is a change of basis matrix if and only if it is nonsingular.

This is because

$$[\mathrm{id}]_C^B[\mathrm{id}]_B^C = [\mathrm{id} \circ \mathrm{id}]_B^B = [\mathrm{id}]_B^B = I_n.$$

- In particular, every elementary matrix is a change of basis matrix.
- Note that $[id]_C^B = ([id]_B^C)^{-1}$.

Let $f\colon\mathbb{R}^2\to\mathbb{R}^2$ be given by f(x,y)=(x+y,2x+2y) and let $B=\{(1,0),(0,1)\}$ and $C=\{(1,2),(1,-1)\}$ be basis for \mathbb{R}^2 . Use the change of basis matrices $[\mathrm{id}]_B^C$ and $[\mathrm{id}]_C^B$ to compute $[f]_C^C$ given that $[f]_B^B=\begin{bmatrix}1&1\\2&2\end{bmatrix}$.