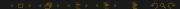
Math 2130 Linear Algebra Week 2 Describing sets of points

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2025 August 29



Today's topics

Describing sets of points

Describing the solution set

Any consistent linear system's solution set has the form

$$\{p + c_1\beta_1 + \cdots + c_k\beta_k \mid c_1, \ldots, c_k \in \mathbb{R}\}$$

where p is any particular solution, k is the number of free variables that system has after Gaussian reduction, and the β_i are column vectors.

Vectors

- We may think of a vector in \mathbb{R}^n as consisting of a magnitude (or length) and a direction.
- Linear equations may represent lines, planes, or their higher-dimensional analogues.

Describing sets of points

Describe the set of points on the line through (1,1) and (2,3). Does the point (4,5) lie on this line? What about (5,7)?

Lines

 \blacksquare The points on the line containing $u,v\in\mathbb{R}^n$ are

$$\{ u + t(v - u) \in \mathbb{R}^n \mid t \in \mathbb{R} \}.$$

Describing sets of points

Describe the set of points on the plane through (1,0,0), (1,1,0), (1,1,1). Does this plane pass through the origin?

Planes

 \blacksquare The points on the plane containing $u,v,w\in\mathbb{R}^n$ are

$$\{ u + s(v - u) + t(w - u) \in \mathbb{R}^n \mid s, t \in \mathbb{R} \}.$$

Planes

■ The points on the hyperplane containing $u_1, u_2, \ldots, u_k \in \mathbb{R}^n$ are

$$\{ u_1 + s_1(u_2 - u_1) + \dots + s_{k-1}(u_k - u_1) \in \mathbb{R}^n \mid s_1, \dots, s_{k-1} \in \mathbb{R} \}.$$

- The set of solutions to a system of linear equations is always such a hyperplane.
- The number of free variables is the dimension of the hyperplane.

Vectors

- We may think of a vector in \mathbb{R}^n as consisting of a magnitude (or length) and a direction.
- Linear equations may represent lines, planes, or their higher-dimensional analogues.