Math 2130
Linear Algebra
Week 2
Describing sets of points

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Today's topics

1. Describing sets of points

Describing the solution set

- Any consistent linear system's solution set has the form

$$\{\, p + c_1\beta_1 + \cdots + c_k\beta_k \mid c_1, \ldots, c_k \in \mathbb{R} \,\}$$

  where p is any particular solution, k is the number of free variables that system has after Gaussian reduction, and the β_i are column vectors.

Vectors

- We may think of a vector in $\mathbb{R}^n$ as consisting of a magnitude (or length) and a direction.
- Linear equations may represent lines, planes, or their higher-dimensional analogues.

Describing sets of points

Describe the set of points on the line through $(1, 1)$ and $(2, 3)$. Does the point $(4, 5)$ lie on this line? What about $(5, 7)$?

Lines

- The points on the line containing $u, v \in \mathbb{R}^n$ are

$$\{\, u + t(v - u) \in \mathbb{R}^n \mid t \in \mathbb{R} \,\}.$$

Describing sets of points

Describe the set of points on the plane through $(1, 0, 0)$, $(1, 1, 0)$, $(1, 1, 1)$. Does this plane pass through the origin?

Planes

- The points on the plane containing $u, v, w \in \mathbb{R}^n$ are

$$\{\, u + s(v - u) + t(w - u) \in \mathbb{R}^n \mid s, t \in \mathbb{R} \,\}.$$

Planes

- The points on the hyperplane containing $u_1, u_2, \ldots, u_k \in \mathbb{R}^n$ are

$$\{\, u_1 + s_1(u_2 - u_1) + \cdots + s_{k-1}(u_k - u_1) \in \mathbb{R}^n \mid s_1, \ldots, s_{k-1} \in \mathbb{R} \,\}.$$

- The set of solutions to a system of linear equations is always such a hyperplane.
- The number of free variables is the dimension of the hyperplane.

Vectors

- We may think of a vector in $\mathbb{R}^n$ as consisting of a magnitude (or length) and a direction.
- Linear equations may represent lines, planes, or their higher-dimensional analogues.