Math 2130
Linear Algebra
Week 0
Introduction to linear algebra

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Welcome to Math 2130

Although the branches of modern mathematics are numerous and varied, it is not unreasonable to claim that two sets of tools are still fundamental to essentially all math, pure or applied. One of those (calculus) should already be familiar to you. In this course we introduce the other: linear algebra.

Welcome to Math 2130

I'll go over the course syllabus, which will also be posted on the 2130 webpage.

Today's topics

1. Linear systems
2. Gauss's method

Linear systems

Definition

An $m \times n$ *system of linear equations* is a collection of m equations of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

where the *system coefficients* a_{ij} and the *system constants* b_j are fixed scalars and the x_i are the unknowns of the system.

Linear systems

Definition

A *solution* to an $m \times n$ system of linear equations is an ordered n-tuple of scalars $(c_1, c_2, \ldots, c_n)$ such that for each i we have

$$a_{i1} c_1 + a_{i2} c_2 + \cdots + a_{in} c_n = b_i.$$

The set of all solutions to a system is called the *solution set* of the system.

Linear systems

- Consider the system

$$
\begin{aligned}
x_1 + x_2 &= 3 \\
3x_1 - 2x_2 &= -1.
\end{aligned}
$$

- We have that $(1, 2)$ is a solution to this system since

$$1 + 2 = 3 \text{ and } 3(1) - 2(2) = -1.$$

- We may also write $x_1 = 1$ and $x_2 = 2$ to indicate this solution.

Linear systems

- Given a general system of linear equations there are a few basic questions we'd like to be able to answer.
  1. Does this system have a solution?
  2. If so, how many solutions are there?
  3. How can we determine all the solutions?
- We'll consider a couple special cases in order to illustrate what the answer to the first two of these questions should be.

Linear systems

- Consider a typical system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

  of two equations in two unknowns.
- This system defines two lines in the (x_1, x_2)-plane.
- These lines are either
  1. parallel and distinct (no solutions),
  2. intersecting at a single point (unique solution), or
  3. the same line (infinitely many solutions).

Linear systems

- A system of three equations in three unknowns defines three planes in space and a similar situation occurs: there are either 0, 1, or infinitely many solutions.

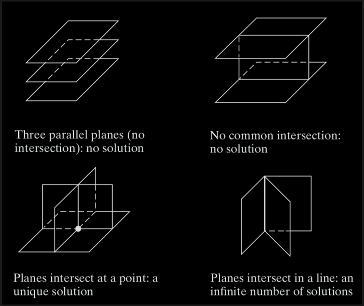

Figure 2.3.1 on page 140 of Differential Equations and Linear Algebra, 4th edition by Stephen W. Goode and Scott A. Annin

Linear systems

- When a system has at least one solution we say that it is *consistent*, otherwise we say it is *inconsistent*.
- We thus seek to determine whether a system is consistent and then, if it is, what its solution set is.